Homework 5

A. Chorin

February 18, 2005

Due Wed. March 2

1. In the last problem of the last set we discretized a Wiener integral by approximating the BM's by walks with n Gaussian increments. Write the solution of this discretized problem as an n-fold ordinary integral. (We shall eventually see how to evaluate such n-fold integrals, even for n large, by efficient Monte-Carlo algorithms).

2. Find the Fokker-Planck equation for the process that satisfies the equation: $du = -dt + dw$ where w is Brownian motion. Does the pdf ever settle to a steady state?

3. Find a stochastic equation whose Fokker-Planck equation is $W_t = 5W + 5xW_x + 16W_{xx}$.

4. Consider particles moving in the plane, with coordinates that satisfy the pair of stochastic equations $dx_1 = a_1 dt + dw_1, dx_2 = a_2 dt + dw_2$, where a_1, a_2 are constants and dw_1, dw_2 independent BM's. The density function $W = W(x, y, t)$ is the joint density of x_1, x_2; find the partial differential equation (Fokker-Planck equation) that it satisfies.

5. Devise a stochastic method for solving the ordinary differential equation $u' = u^2$, $u(0) = 0.5$. Hints: branch, and note that $u^2 = -u + (u^2 + u)$. Use your method to determine $u(1)$ on the computer, and compare the result with the exact solution.

6. Prove that $e^{s\Delta}e^{t\Delta} = e^{(s+t)\Delta}$, where $\Delta = \partial^2/\partial x^2$ (You first have to figure out what this means, and then check by means of formulas).

7. Evaluate $e^{t\frac{\partial}{\partial x}} f$ for $f = \sin x$, at the point $x = 1, t = 1$.

.